Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS: SPECIALIST

UNITS 3A AND 3B

FORMULA SHEET
2012

This document is valid for teaching and examining until 31 December 2012.

Vectors

$|(a_1, a_2)| = \sqrt{a_1^2 + a_2^2}$ $\qquad |\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta = a_1 b_1 + a_2 b_2$

Vector equation of a line in the plane:

one point and the slope:	$\mathbf{r} = \mathbf{r}_1 + \lambda \mathbf{l}$
two points:	$\mathbf{r} = \mathbf{r}_1 + \lambda(\mathbf{r}_2 - \mathbf{r}_1)$
normal:	$\mathbf{r} \bullet \mathbf{n} = c$

Vector equation of a circle in the plane: $|\mathbf{r} - \mathbf{d}| = \rho$

Trigonometry

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

$$\text{Length of arc} = r\theta$$

$$\text{Area of segment} = \frac{1}{2} r^2 (\theta - \sin\theta)$$

$$\text{Area of sector} = \frac{1}{2} r^2 \theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$$

$$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$$

$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta\tan\varphi}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

See next page

Exponentials and logarithms

For $a, b > 0$ and m, n real:

$a^m a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

For $a > 0$ and m an integer and n a positive integer:

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

For a, y, m, n positive and real and k real:

$1 = a^0 \Leftrightarrow \log_a 1 = 0$

$y = a^x \Leftrightarrow \log_a y = x$

$\log_a(mn) = \log_a(m) + \log_a(n)$

$a = a^1 \Leftrightarrow \log_a a = 1$

$\log_a(m^k) = k \log_a(m)$

Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height

Prism: $V = Ah$, where V is the volume, A is the area of the base and h is the perpendicular height

Pyramid: $V = \frac{1}{3}Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

See next page

Functions

If $f(x) = y$ then $f'(x) = \dfrac{dy}{dx}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

If $f(x) = e^x$ then $f'(x) = e^x$

If $f(x) = \ln x$ then $f'(x) = \dfrac{1}{x}$

Product rule:

If $y = f(x)\, g(x)$

then $y' = f'(x)\, g(x) + f(x)\, g'(x)$

or

If $y = uv$

then $\dfrac{dy}{dx} = \dfrac{du}{dx} v + u \dfrac{dv}{dx}$

Quotient rule:

If $y = \dfrac{f(x)}{g(x)}$

then $y' = \dfrac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$

or

If $y = \dfrac{u}{v}$

then $\dfrac{dy}{dx} = \dfrac{\dfrac{du}{dx} v - u \dfrac{dv}{dx}}{v^2}$

Chain rule:

If $y = f(g(x))$

then $y' = f'(g(x))\, g'(x)$

or

If $y = f(u)$ and $u = g(x)$

then $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Quadratic function:

If $y = ax^2 + bx + c$ and $y = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $x \in \mathrm{C}$

Piecewise-defined functions:

Absolute value function: $|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Sign function: $\operatorname{sgn}(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

Greatest integer function:
$\operatorname{int}(x)$ = greatest integer $\leq x$ for all x

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.